## BRINGING MINDFULNESS INTO HIGHER EDUCATION *by Richard S. Ellis*

The benefits of meditation are apparent to everyone who practices this ancient technique. Meditation calms the mind and brings equanimity. It enables us to connect with the wisdom of our bodies and the wisdom of the present moment. It also helps us cope with pain, reduce stress, and alleviate suffering. The insight gained from meditation is called mindfulness. This is the calm and direct awareness of what is happening in the present moment, in your body, in your mind, and in the world around you. By focusing your attention on the present moment, mindfulness cultivates wakefulness and wisdom.

After meditating regularly for a number of years, I decided to introduce meditation and the benefits of mindfulness, in a gentle and nonintrusive way, to the students I teach. Starting five or six years ago and continuing into the present, I have begun each class with a short meditation exercise, which I do without using the language of meditation and which I make clear is completely voluntary. With small variations here are my instructions.

> *If you would like to participate, then I invite you to close your eyes, sit up straight in the chair, and start breathing slowly. Just relax. Give yourself the gift of doing nothing but breathe. As you become aware of your breath, start to feel present.*

I also discuss issues of stress with the students. I know that the combination of meditation and discussion is working. For example, in the 20 course evaluations in an undergraduate course that I taught in the fall of 2012, 10 students commented favorably on the meditation exercise or remarked that I had created a low-stress learning environment. "I thought the meditation sessions were helpful and helped me to focus better during class," wrote one student. Another remarked, "Overall, even though this class is one of the more difficult ones, it was by far my least stressful."

During the spring semester of 2013 I wanted to go further, building on an article about stress in academic life that was published in last year's newsletter. First I spoke with my Department Head, **Michael Lavine**, and with two Associate Deans in my college, all of whom strongly supported me in this endeavor. Their expressions of support inspired me to organize a group of graduate students in my department with whom I meet once a week to discuss issues of stress and to meditate together, cultivating the mindfulness that can heal that stress. I can see in their faces that our discussions are having a profound impact. These graduate students, having grown up, like me, in a culture of competition and overwork, welcome the wisdom of mindfulness, which teaches that the

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## b squared minus the square root of b plus or minus 2 over 4ac ... or Something Like That

*by Jeff Beaulieu*

Among my four hundred students during the past spring semester, there was a sixth grader aspiring to be an endocrinologist, there was someone who graduated with me from high school and reminded me that our twenty-fifth class reunion is taking place this year, and there were roughly fifteen to twenty students registered as learning disabled, some of whom I met with regularly outside of normal office hours. This was a typical semester, though by no means do I consistently have middle-schoolers or high school classmates as students in my classes, but it's interesting when I do, and it's interesting that I can.

Teaching large enrollment courses at the university provides an opportunity to get to know people—not everyone, but I get to know some students very well. Teaching introductory courses in mathematics ensures that I get to know many students' fears and dislikes for the subject matter, as they are often willing to disclose this information, unsolicited, as a defense against learning the dark art of algebra. I don't dismiss students' fears and dislikes, but I don't dwell on them, and I discourage them from doing the same. Acknowledging their apprehension toward mathematics can be helpful, but then so is putting it aside to get down to the business of finding out why these learning barriers exist, and how to get past it.

My belief is that students need to feel that they can do mathematics. In short, it's a confidence problem. Somewhere in their mathematics education, they got stuck, hit a wall, or simply became disinterested. A common thread seems to be that many students are missing a piece of helpful information to bridge the gap between two concepts, some little tidbit that makes it all make sense. These students don't need to learn the answers to life's most challenging questions—which is good for me, because I don't have them—but I feel pretty confident that I can give them the tools they need for, say, solving quadratic equations.

When I ask students if they remember the quadratic formula, typically I hear the groans that indicate definite familiarity with the phrase, coupled with slight annoyance. These reactions are probably because it is one of the most excruciating formulas they had to memorize in high school. As the title of this article suggests, the formula is long, has lots of letters and a square root, and what the heck is that plus or minus thing anyway? If asked to scribble it out on paper, most would have some memory of its structure, but only a few would have it exactly correct. One of the joys of my job is showing students where these formulas come from and why they're useful. So, having already solved quadratic equations by completing the square, we then go through the process of completing the square on the general quadratic equation $ax^2 + bx + c = 0$. It's not easy for them because we're dealing with a lot of

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pressures of academic life are unavoidable but that stress can end. Next year I would like to expand this gathering to include students from other departments.

In their evaluations all the graduate students who participated indicated that they found the weekly meetings extremely helpful. One student elaborated on this when he wrote, "At these meetings, I have been able to practice and explore mindfulness meditation under the tutelage of a knowledgeable and experienced mentor, Professor Ellis. I find the meditation calming, and the short discussions the group engages in prior to meditating are always lively and thoughtful."

I am excited about the possibility of sharing the benefits of mindfulness with more people at the university. I look forward to helping them experience, as I have, how mindfulness can heal the suffering caused by the pressures of academic life and can transform that suffering into insight and wisdom.

**Annual Group Project in Applied Mathematics** *continued from page 17*

Algorithms and Parallel Processing.

The second subgroup of students tackled the timely topic of swarming behavior, in which the agents are meant to model fish in a school or birds in a flock. Remarkably, these self-organizing behaviors in biology can be modeled by merely adding one or two terms to the equations of motion for molecular dynamics. One retains the interaction potential to give the swarm a preferred separation distance, and one adds either a self-propulsion term or a velocity alignment term to the acceleration equation. The celebrated Cucker-Smale model introduces only the velocity-alignment effect, and its dynamics are quite simple: for strong local alignment of velocity, any initial condition (random positions and velocities) will eventually develop into a coherent motion in which all the agents have the same velocity. If one now adds attractions and repulsions and perhaps an external force to simulate avoidance of a moving point, representing, for example, a predator on the prowl, then the dynamics becomes interesting, even surprising. The students went one step further by introducing a cone of vision for each agent, which limits the velocity-alignment effect to a forward-moving window of sight; each agent tries to adjust to the other agents within its cone of vision only. What results is fascinating collective behavior, with recurrent spirals of the agents that undulate, partially disperse, and then recombine into coherent patterns. In this case a computer movie is worth a thousand words, and the videos presented at the colloquium got quite a reaction from the audience.

Another modification was to include a self-propulsion effect without imposing full velocity alignment. In that model the added term pushes the agents toward having a common speed, but is independent of direction. Nonetheless, swarming behavior again results. In this case a "windmill" pattern can emerge from a generic initial condition, in the sense that a seemingly chaotic swarm will rather suddenly make a transition to a nearly coherent rotational pattern without provocation. Such models have been used in contemporary computer graphics, and in fact the stampede of the wildebeests in the movie "The Lion King" used such simulation techniques. Inspired by this, one of the students could not resist the temptation to create a model canyon, defined by appropriate external potentials and with obstacles placed along the path, and to force a herd of wildebeests through it according to the model equations. Remarkably realistic effects are produced as the herd gathers and separates to move over the imposed topography. Mathematics at the movies!

The third subgroup of students expressed interest in studying a model from finance, and so they probed the literature on multi-agent models of stock markets. In this case the agents are traders who put buy and sell orders into the market based on their expectations of the future performance of a stock. The students decided to build a deterministic model of the clearinghouse in which such buying and selling occurs. Their approach was more realistic than many academic models, which ignore some of the details of how orders are cleared in real time and instead introduce some randomness to account for the neglected mechanisms. Using an algorithmic clearinghouse with two stocks, the students simulated the dynamics of the stock prices due to the activity of 30 traders. These agents were classified into three different trading strategies, or styles: long-term investors, momentum traders, and intraday speculators. A very satisfying result of their multi-agent model was its ability to replicate the qualitative behavior of real stock prices on the time scale of hours to 5 days, and in particular to show that the pseudo-random fluctuations seen in these markets can be the result of interactions between many agents who are buying and selling according to non-random rules. Only the timing of when orders are placed or filled was randomized in their model. When the intraday speculators were removed from the simulated market, however, it was found to go dead. This result suggests that certain trading styles may be important to maintaining the typical short-term fluctuations of stock prices. Their presentation ended with several provocative implications of this kind, and left everyone wondering what other insights might be revealed by further investigations with such models.

As it has in previous years, the group project was a lot of work and a lot of fun. This year the results relied heavily on computationally intensive simulations, and the fascinating collective behaviors of the multi-agent models could not have been revealed without those computations. In this way the project reflected a powerful trend in contemporary applied science, the union of mathematical and statistical modeling with massive computations.